Introduction to Decision Deck-Diviz: Examples and User Guide

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1 - Introduction

Humans face daily complex decision making problems in many diverse fields. This issue is especially critical in business or governmental decisions, because it may involve large amounts of money. The frequency and complexity of decision making requires the development of multidisciplinary methods that help the decision maker.

The area of Multiple Criteria Decision Aid (MCDA) is focused on providing solutions to decision problems where many criteria and preferences must be taken into consideration. Many important technical aspects of MCDA are linked to classic works in economics, in particular, welfare economics, utility theory and voting oriented social choice theory. Aggregating the opinion or the preferences of voters or individuals of a community into collective or social preferences is quite similar a problem to devising comprehensive preferences of a decision-maker from a set of conflicting criteria in MCDA.

The literature in the field of Multi-criteria Decision Aid has proposed a large number of methodologies that are acknowledged in the research community and used in real world decision processes. These methodologies provide useful solutions to complex decision problems that arise in organizations. Several methods have been implemented independently during the years in an uncoordinated way, using different language and software tools. This software is rarely available, or it has become obsolete because it is not maintained.

The Decision Deck project aims at collaboratively developing Open Source software tools implementing a platform composed of modular and interconnected software components. These software components implement the common functionalities of a large range of MCDA methods.

In this document we will review the Decision Deck project and its main components. We will focus on the *Diviz* module, which is the newest one, and it is particularly designed for academic purposes, as well as, to facilitate the reuse and comparison of MCDA modules. Two different methodologies will be analysed in detail:

the Weighted Sum Model (based on Utility Theory) and ELECTRE III (based on Outranking). User guide indication will be given, in order to facilitate the use of *Diviz* to test those methods with different data sets.

2 – Overview of the Decision Deck project

The Decision Deck project aims at collaboratively developing Open Source software tools implementing Multiple Criteria Decision Aid (MCDA). Its purpose is to provide effective tools for three types of users:

- Practitioners who use MCDA tools to support actual decision makers involved in real world decision problems;
- Teachers who present MCDA methods in courses, for didactic purposes;
- Researchers who want to test and compare methods or to develop new ones.

As stated before, the goal of the Decision Deck project is to offer an integrated platform for using the available MCDA methods developed, facilitating the testing and comparison of those methods. In addition, they offer tools to incorporate new decision methods, whatever language they are implemented, without having to rewrite them. This is achieved by means of Decision Deck plug-in programming, which must fulfil some specification requirements. This design makes possible to easily integrate additional methods without great effort.

To achieve these goals, the Decision Deck project works on developing multiple software resources that are able to interact. Along the history of the project, different tools have been developed (1):

- **XMCDA**: a standardized XML recommendation to represent objects and data structures issued from the field of MCDA. Its main objective is to allow different MCDA algorithms to interact and be easily callable.
- **XMCDA web services**: distributed open source computational MCDA resources, using the XMCDA standard.
- **DIVIZ**: an open source Java client and server for designing, executing and sharing MCDA methods, via the composition of XMCDA web services.
- **D^2**: a rich open source Java software containing several MCDA methods.

- **D^3**: an open source rich internet application for XMCDA web services management.

In order to coordinate the various activities of the Decision Deck project, it is structured as follows:

- **The Decision Deck Consortium**: a French non-profit association which steers and manages the project along the lines of this manifesto. It is headed by an administration board. The consortium is among other in charge of organising the workshops of the project every semester.
- **The Software Resources Management Groups**: they are in charge of the organisation and the management of the developments of the six identified initiatives of the Decision Deck project. Each group is coordinated by a clearly identified contact person. These management groups are in charge of organising the bi-annual developers' days of the project.
- **The Specifications Committee**: under the direction of a coordinator, its role is to maintain and develop the XMCDA standard and to approve and publish suggested evolutions. The coordinator is in charge of organising the specifications meetings.
- **The Communication & Dissemination Committee**: under the direction of a coordinator, its role is to develop and maintain the websites of Decision Deck and to manage the communicational aspects of the project.

2.1 - XMCDA

XMCDA is a data standard which allows representing Multi Criteria Decision Analysis (MCDA) data elements in XML according to a clearly defined grammar.

XMCDA is an instance of UMCDA-ML, which is the Universal Multi Criteria Decision Analysis Modelling Language and which is one of the scientific initiatives inside the Decision Deck project. UMCDA-ML is intended to be a universal modelling language to express MCDA concepts and generic decision aid processes.

XMCDA focuses particularly on MCDA concepts and data structures and is defined by an XML schema.

XMCDA is maintained by the Decision Deck project and supported by the COST Action IC0602 Algorithmic Decision Theory.

The goals of XMCDA are to ease:

- The interaction of different MCDA algorithms.
- The execution of various algorithms on the same problem instance.
- The visual representation of MCDA concepts and data structures via standard tools like web browsers.

A very natural field of application of XMCDA is given by the web services developed inside the Decision Deck project.

2.2 - Decision desktop (D^2)

The Decision Desktop software, or D^2 for short, was the first software to be developed in the Decision Deck project.

It is a desktop, client/server application, meaning that it is designed to be installed locally (it is not a web application), and uses a database to store application data, thereby enabling a multiple user usage.

One of its usage patterns is that several experts may enter evaluations in a decentralized manner, then these evaluations are analysed by a coordinator, this analysis is reviewed by one or several decision-makers.

Three hard-coded roles have been identified:

- Coordinator: The Coordinator has the responsibility to specify the model.
- Decision Maker: The Decision Maker has the responsibility to choosing an alternative.
- Evaluator: The evaluator has the responsibility to evaluate alternatives

Historically the software has also been named EVAL, from the name of the originating project, and Decision Deck. It is now recommended to name it decision desktop to differentiate it from the other software produced in the Decision Deck project.

D^2 offers several MCDA methods, like:

- **Iris** is an interactive method that aims at sorting alternatives into ordered classes. The assignment of alternatives to classes corresponds to ELECTRE Tri. The specificity of the IRIS method is that the decision maker does not have to specify the weights of criteria, but instead provides assignment examples, i.e., typical elements of the classes (2) (3) (4).
- **Rubis** is a decision aid method for tackling the choice problem in the context of multiple criteria decision analysis (5).

  Its genuine purpose is to help a decision maker to determine a single best decision alternative. Methodologically we focus on pairwise comparisons of these alternatives which lead to the concept of bipolar-valued outranking digraph (6). The work is centred around a set of five pragmatic principles which are required in the context of a progressive decision aiding methodology (7).
- The **VIP** (Variable Interdependent Parameters) Analysis software has been built to support the selection of the most preferred alternative among a list, considering the impacts of each alternative on multiple evaluation criteria. It is based on an additive aggregation model (value function), accepting imprecise information on the value of the scaling coefficients (a.k.a. scaling constants, which indirectly reflect the relative importance of the each criterion) (8).
- The **UTAGMS** and **GRIP** methods aim at solving the multiple criteria ranking problem. The underlying preference model is a set of monotone additive value

function compatible with preference statements expressed by the decision maker. The main result of these methods is:

  - a necessary (robust) ranking that contains comparisons of alternatives that remain valid for all compatible value function,
  - a possible ranking that contains comparisons of alternatives that is valid for at least one compatible value function.

  For detailed descriptions of the methods, see:

  - UTAGMS (9).
  - GRIP (10).

- Weighted Sum.

One of the advantages is that we do not need Internet to work because the user has a local database to store all its methods. However, this complicates the sharing of work.

2.3 - Distributed Decision Deck (D^3)

D^3 is a distributed and collaborative approach to Multi Criteria Decision Aid. It is a Rich Internet Application that allows interacting with remote web services exposing MCDA methods. They have a served (Distributed Decision Deck) resource that permits to integrate such existing methods. Data exchanges between D^3 and remote MCDA services are done through XML files using a SOAP literal RPC protocol.

In the previous section we have seen that the user must have its own database, which stores all its methods, this is a drawback, because nobody else can use them. The distributed version, as just discussed, provides the ability to share methods among the community, forcing to use Internet.

2.4 - Diviz

Diviz enables to conveniently combine programs implementing MCDA algorithms in a modular way, decomposing them into subsystems. It is an easy tool to build, execute and share complex workflows of MCDA algorithms. In the literature, such workflows are often called methods (11).

In this section we present the main features of *Diviz* and show how it allows rebuilding classical MCDA methods, to develop new ones by combining various elementary components, and how it can be used as a research and dissemination tool.

2.4.1 - Workflow design

The design of the MCDA workflows is performed via an intuitive graphical user interface, where each algorithm is represented by a box that can be linked to data files or supplementary calculation elements by connectors. Thus, the design of complex algorithmic workflows does not require any programming skills, but only needs understanding the functioning of each calculation module.

To construct a new MCDA workflow, the user chooses one or more modules from a list of available calculation elements that he can drag and drop in a dedicated workspace. Then he adds data files to the workspace and connects them appropriately to the inputs of the elements. Finally, the user connects the inputs and outputs of the components by connectors to define the structure of the workflow.

2.4.2 - Execution and results

Once the design of the MCDA workflow is finished, it is possible to execute it in order to obtain the output values of the algorithms. In *Diviz*, these calculations are performed on computing servers through the use of web-services published by the Decision Deck project. The idea behind these web-services is to allow anyone who is connected to the Internet to access a large amount of MCDA algorithms without having to install them on their personal computer. As a consequence, *Diviz* does not contain any algorithmic modules, but requires a connection to the Internet to access the calculation resources.

If the execution of the workflow is successful, the outputs of each of the components can be viewed and analysed by the user. Some of these outputs might represent results of intermediate calculation steps of the decision aid workflow that has been built by the user.

Furthermore, the history of the past executions is kept in *Diviz* and can be viewed by the user. In particular, if a workflow is modified, the former execution'

results and their associated workflows are still available. Again this is a key feature which helps calibrating the parameters of the algorithms which are used in a workflow, as any past execution can be recalled and reanalysed.

2.4.3 - Workflow sharing

The *Diviz* software enables to export any workflow, with or without the data, as an archive. The workflow can then be shared with any *Diviz* user, who can then import it (by loading the archive) into his software and continue the development of the workflow or execute it on the original data.

Consequently, *Diviz* is especially interesting for academic purposes, because the teacher can prepare workflows that can be used by the students. In the same way, the students' workflows and results can be evaluated by the teacher.

In addition, *Diviz* provides some interesting tools for research, as the results or workflows obtained with these tools can become supplementary electronic material to attach to research papers.

2.4.4 – Installation

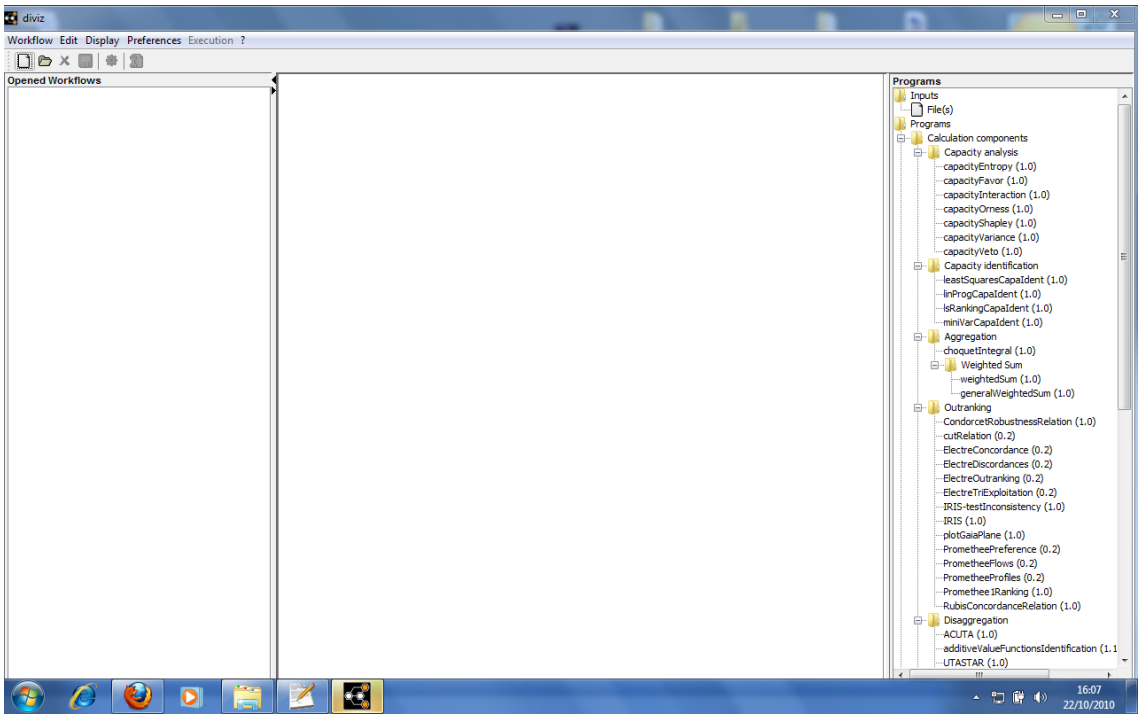
To use *Diviz*, you must download the desktop software, *Diviz* requires to have Java (JRE 6 Update 16 (or later)) installed on your computer. Once the software is installed, we only need an Internet connection to start to use it.

To run *Diviz*,

- either double click on the downloaded jar file;
- or run the following command (in a terminal window (Linux) or a command prompt (Windows)): **"java -jar diviz-client.jar"**

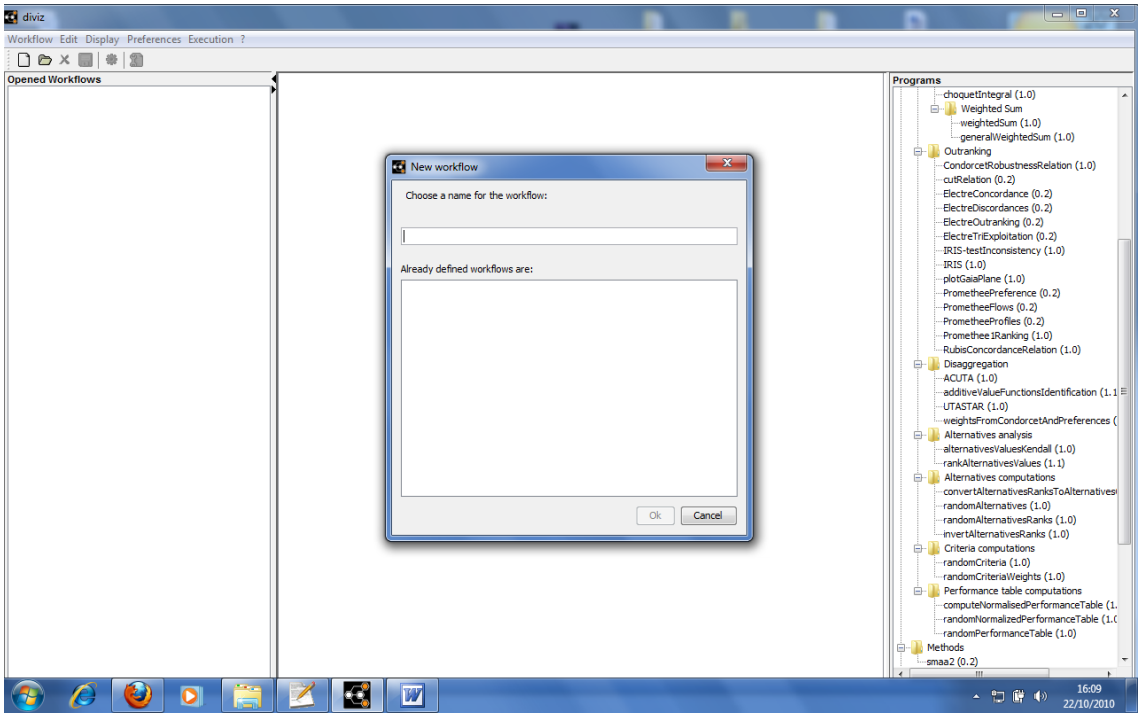
For Windows users to open the command prompt, click Start, point to All Programs, point to Accessories, and then click Command Prompt.

Now, we will show some snapshots of *Diviz* in order to illustrate the basics of the *Diviz* software.

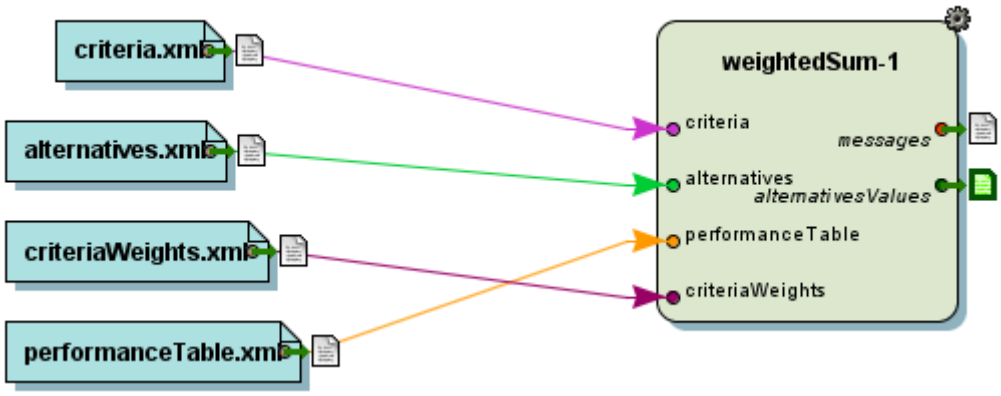


To start to work we need to create a new workflow, so that we select in the menu:

- Workflow → New



Once created the workflow, we can begin to put the blocks we want to build our decision support system. For example we could build the next model:



The boxes on the left indicate the data files that contain the input information about the decision problem. The box on the right is a module that is associated to some web-service. So it is ready to be executed and it applies the Weighted Sum MCDA method to the data introduced. Results can be seen by clicking on the *alternativesValues* square on the right side of this block.

As it will be seen in the next sections, it is possible to connect some plotting modules to the output of the MCDA methods in order to obtain a graphical representation of the results.

3 - Three Basic Concepts

First of all, and for the sake of clarity of this document, in this chapter we define the basic concepts of MCDA problems, following the nomenclature in (12): alternatives, criteria and decision problem.

3.1 – Alternative

The general concept of *action* is used to designate that element which constitutes the object of the decision, or that which decision aiding is directed towards. The concept of *action* does not a priori incorporate any notion of feasibility, or possible implementation. An action is qualified as *potential* when it is deemed possible to implement it, or simply when it deserves some interest within the decision aiding process.

The concept of *alternative* corresponds to the particular case in which modelling is such that two distinct potential actions can in no way be conjointly put into operation. This mutual exclusion comes from a way of modelling which in a comprehensive way tackles that which is the object of the decision, or that towards which decision analysis is directed. Many authors implicitly suppose that potential actions are, by definition, mutually exclusive. Although this hypothesis is in no way compulsory, we will assume it in the rest of the document.

In all cases, A will denote the set of alternatives considered at a given stage of the decision process. By a, we will denote any single alternative. In this document we will assume that the number of actions is finite $A = m$ having:

$$A = a_1, a_2, \dots, a_m$$

3.2 – Criterion

A criterion c is a tool constructed for evaluating and comparing alternatives according to a particular point of view. This evaluation must take into account, for each action a, all the pertinent effects or attributes linked to the point of view considered. Sometimes criteria can be understood as goals to be achieved by the best alternative.

A criterion is denoted by $c\ a$ and called the *performance* of the alternative *a* according to some aspect of the problem. This performance can be interpreted as the degree of satisfaction of the decision maker with respect to a certain variable (feature, property).

The set of criteria will be always finite $C\ = p$ and denoted as:

$$C = \ c_1, c_2, \dots, c_p$$

The scale of measurement of a criterion can be numerical or qualitative. For more information on data types, we refer the reader to (13).

3.3 – Decision Problem

In MCDA, the definition of the type of decision problem deals with answers to questions such as the following (12):

1. In what terms should we pose the problem?
2. What type of results should we try to obtain?
3. How does the analyst see himself fitting into the decision process to aid in arriving at these results?
4. What kind of procedure seems the most appropriate for guiding his investigation?

Decision problems are traditionally classified into three types as follows:

- The *choice* problematic: The aid is oriented towards and lies on a selection of a small number (as small as possible) of "good" actions in such a way that a single alternative may finally be chosen. This does not mean that the selection is necessarily oriented towards the determination of one or all the actions of *A* which can be regarded as optimum. The selection procedure is based on comparisons between actions so as to justify the elimination of the greatest number of them, the subset of those actions which are selected (which can be viewed as a first choice) containing all the most satisfying actions, which remain non comparable between one another.

- The *sorting* problematic: The aid is oriented towards and lies on an assignment of each action to one category (judged the most appropriate) among those of a family of predefined categories, which are usually ordered according to some preference consideration. This family of categories must be conceived on the basis of the diverse types of treatments that motivate the sorting. For instance, a family of four categories can be based on a comprehensive appreciation leading to distinguishing between: actions for which implementation (i) is fully justified, (ii) could be advised after only minor modifications, (iii) can only be advised after major modifications, (iv) is unadvisable.
- The *ranking* problematic: The aid is oriented towards and lies on a complete or partial preorder on *A* which can be regarded as an appropriate instrument for comparing actions between one another; this preorder is the result of a classifying procedure allowing us to put together in classes actions which can be judged as indifferent, and to rank these classes (some of them may remain non-comparable).

4 - XMCDA definitions

This chapter presents the main features of the language *XMCDA*, a standardised *XML* proposal to represent objects and data issued from the field of Multiple Criteria Decision Aid (14). Its main objective is to allow different MCDA algorithms to interact using a common nomenclature, as well as to be easily callable from other software components like the *Diviz* platform of the Decision Deck project. In fact, the definition of XMCDA is part of the Decision Deck project.

The language is structured in sets of tags to identify the different MCDA concepts. All the tags appear into < >. The following subsections give the details of the basic tags that are needed to construct the data files that are used in *Diviz*.

4.1 – Names of the tags

By convention, the name of a tag starts by a lower-case letter. The rest of the name is in mixed case with the first letter of each internal word capitalised. This allows easily reading and understanding the meaning of a tag. We use whole words and avoid as much as possible acronyms and abbreviations. Consider for example the tag names *methodOptions*, *performanceTable* and *criterionValue*. Note that objects of the same XMCDA type can in general be gathered in a compound tag, represented by a single XML tag named after the plural form of its components (e.g., **alternatives**).

4.2 Attributes of the tags

The three following attributes can be found in any of the main data tags: *id*, *name* and *mcdaConcept*. They are in general optional, except for the id attribute in the description of an alternative, a criterion or a category. Each of these three attributes has a particular purpose in XMCDA:

- The **id** attribute allows the identification of an object with identifier.
- The **name** attribute allows giving a name to a particular object.
- The **mcdaConcept** attribute allows identifying the MCDA concept linked to a particular instance of an XMCDA type.

```
<alternatives mcdaConcept="Congress Hotels Finland">
      <alternative id="a01" name="Jyvaskyla sokos Hotel Alexandra" />
      <alternative id="a02" name="Jyvaskyla sokos hotel Jyvashovi" />
      <alternative id="a03" name="Cumulus Jyvaskyla" />
      <alternative id="a04" name="Scandic Jyvaskyla" />
      <alternative id="a05" name="Hotel Pension Kampus" />
      <alternative id="a06" name="Hotelli Alba" />
</alternatives>
```

4.3 Definition of alternatives

Alternatives are defined and described under the *alternatives* tag. They can be either **active** or not and either be **real** or **fictive** alternatives. In addition, they can also be flagged as **reference** alternatives (for profiles in a sorting problem, e.g.). The **id** of an alternative is mandatory.

```
<alternatives name="myAlternatives">
      <alternative id="x1" name="Red Ferrari"/>
      <alternative id="x2" name="Blue Corvette">
            <type>real</type>
            <active>true</active>
            <reference>false</reference>
      </alternative>
      <alternative id="x3" name="UFO">
            <type>fictive</type>
      </alternative>
</alternatives>
```

4.4 Definition of criteria

Criteria are defined and described under the *criteria* tag. For each criterion one has to define its **id**. In the following example, the first criterion *g1* represents the power of a car.

```
<criteria>
      <criterion id="g1">
            <description>
                  <comment>Power in horsepowers</comment>
            </description>
            <attributeReference>att1</attributeReference>
            <scale>
                  <quantitative>
                        <preferenceDirection>max</preferenceDirection>
                        <minimum><real>50</real></minimum>
                        <maximum><real>200</real></maximum>
                  </quantitative>
            </scale>
      </criterion>
      <criterion id="g2"/>
</criteria>
```

4.4 The performance table

The performance table is defined and described under the tag *performanceTable*. It contains, for each alternative (given by its **id**), a list of performances, given by a criterion **id** and a corresponding performance **value**.

```
<performanceTable>
      <alternativesPerformance>
            <alternativeID>alt1</alternativeID>
            <performance>
                  <criterionID>g1</criterionID>
                  <value><real>72.10</real></value>
            </performance>
            <performance>
                  <criterionID>g2</criterionID>
                  <value><real>82.62</real></value>
            </performance>
      </alternativesPerformance>
      <alternativesPerformance>
            <alternativeID>alt2</alternativeID>
            [..]
      </alternativesPerformance>
</performanceTable>
```

5 - Weighted sum model

The weighted sum model (WSM) is the most well-known and simplest multi-criteria decision method that follows the Multiattribute Utility Theory (MAUT). MAUT is based on the idea that any decision-maker attempts unconsciously to maximize some function that aggregates the utility of each different criterion. So, in this case, the preference values of the criteria are understood and treated as utilities (15).

In this model, each criterion is understood as a partial utility, where c_j is a strictly increasing function that returns values in a common scale, in order to allow criteria to be compared and added without problems with different units of measurement. Once the c_j are known, the MAUT methods consider two steps to be followed (16):

- Aggregation (rating): a global value for each alternative is computed, which gives a general idea of the utility of the alternative considering all the criteria at the same time;
- Exploitation: the utility values obtained in the first step are used to find the best alternative, to rank them or to classify the alternative into some predefined groups.

In the first step, some mathematical operator to aggregate the partial utilities to obtain a global one is required. The simplest one is the weighted, where having a set of m alternatives and p decision criteria, we attach a value w_j denotes the relative weight of importance of the criterion c_j. The set of weights must add 1, that is $\sum_{i=1}^{p} w_i = 1$.

Then, having that x_{ij} is the performance value of alternative a_i when it is evaluated in terms of criterion c_j, the overall performance score (or global utility value) of alternative a_i, denoted as $a_i^{WSM-score}$, is calculated as the weighted average as follows:

$$a_i^{WSM-score} = \sum_{j=1}^{p} w_j x_{ij}, for\ i = 1,2,3,\dots,m.$$

5.1 - Example

For a simple numerical example suppose that a decision problem of this type is defined on three alternatives a_1, a_2, a_3 each described in terms of four criteria c_1, c_2, c_3 and c_4. Furthermore, let the numerical data for this problem be as in the following decision matrix:

	c_1	c_2	c_3	c_4
a_1	20	31	37	11
a_2	85	53	45	76
a_3	49	34	52	19
w	0.20	0.15	0.40	0.25

For instance, the relative weight of the first criterion c_1 is equal to 0.20; the relative weight for the second criterion is 0.15 and so on. Similarly, the value of the first alternative a_1 in terms of the first criterion is equal to 20; the value of the same alternative in terms of the second criterion is equal to 31 and so on.

When the WSM is applied on these numerical data the overall performance scores for the three alternatives are:

$$a_1^{\text{WSM-score}} = 20\text{x}0.20 + 31\text{x}0.15 + 37\text{x}0.40 + 11\text{x}0.25 = 26.2$$

Similarly, one gets:

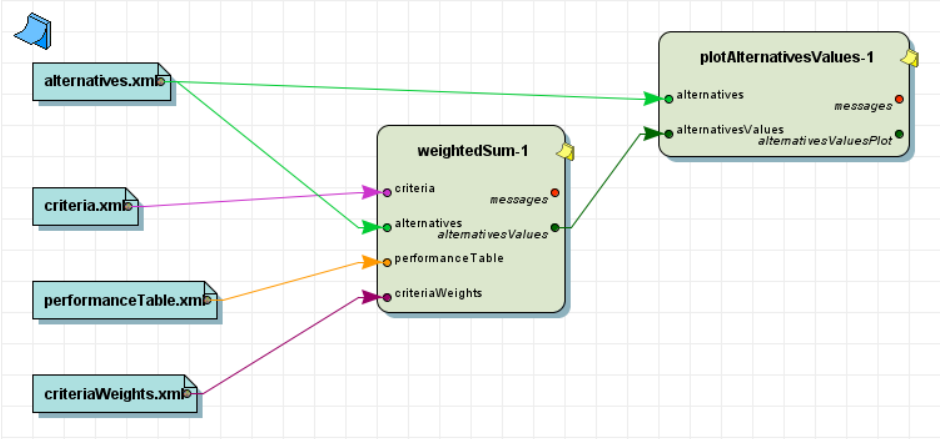
$$a_2^{\text{WSM-score}} = 61.95, \text{and } a_3^{\text{WSM-score}} = 40.45$$

Thus, the best alternative (in the maximization case) is alternative a_2 (because it has the maximum WSM score which is equal to 61.95). Furthermore, these numerical results imply the following ranking of these three alternatives: $a_2 > a_3 > a_1$ (where the symbol ">" stands for *better than*).

5.2 - Example Weighted Sum Model in Diviz

Here, we present the example with *Diviz* software. The first step is to build the workflow, where you enter the 4 data files and the modules *WeightedSum* and *plotAlternativesValues*. The workflow for WSM is the following one. The plot

component permits to have a graphical representation of the overall performance values obtained.



The data files are written using the language XMCDA. The following tables show the code of each data for the example presented in the previous section.

- **Alternatives:**

Code XMCDA	Diviz
<alternatives> <alternative id="a01"> <active>true</active> </alternative> <alternative id="a02"> <active>true</active> </alternative> <alternative id="a03"> <active>true</active> </alternative> </alternatives>	• Id : **a01** *Active : true* • Id : **a02** *Active : true* • Id : **a03** *Active : true*

- **Criterion:**

Code XMCDA	Diviz
<criteria> <criterion id="c01" /> <criterion id="c02" /> <criterion id="c03" /> <criterion id="c04" /> </criteria>	• Id : **c01** • Id : **c02** • Id : **c03** • Id : **c04**

- **Weights:**

Code XMCDA	DIVIZ
<criteriaValues mcdaConcept="weights"> <criterionValue> <criterionID>c01</criterionID> <value> <real>0.20</real> </value> </criterionValue> <criterionValue> <criterionID>c02</criterionID> <value> <real>0.15</real> </value> </criterionValue> <criterionValue> <criterionID>c03</criterionID> <value> <real>0.40</real> </value> </criterionValue> <criterionValue> <criterionID>c04</criterionID> <value> <real>0.25</real> </value> </criterionValue> </criteriaValues>	**weights** c01 0.2 c02 0.15 c03 0.4 c04 0.25

- **Performance Table**:

Code XMCDA: only the first alternative	DIVIZ: Shown all alternatives
<performanceTable> <alternativePerformances> <alternativeID>a01</alternativeID> <performance> <criterionID>c01</criterionID> <value> <integer>25</integer> </value> </performance> <performance> <criterionID>c02</criterionID> <value> <integer>20</integer> </value> </performance> <performance> <criterionID>c03</criterionID> <value> <integer>15</integer> </value>	**Performance table** c01 c02 c03 c04 a01 20 31 37 11 a02 85 53 45 76 a03 49 34 52 19

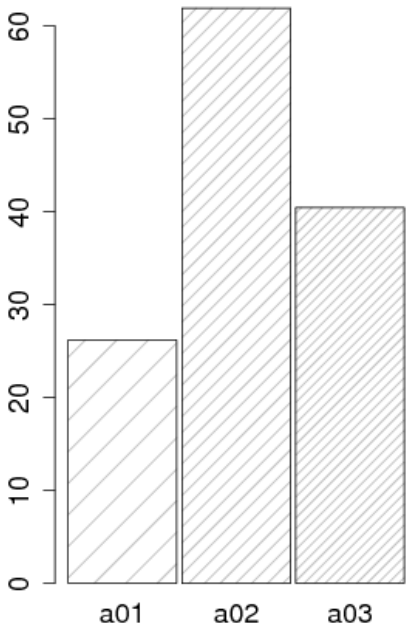
```
      </performance>
      <performance>
         <criterionID>c04</criterionID>
         <value>
            <integer>30</integer>
         </value>
      </performance>
   </alternativePerformances>
</performanceTable>
```

For consulting the complete code of this example see Annex A.

5.3 - Results

After running the method with *Diviz*, the values obtained are given in the following table. Moreover, using the plot module in *Diviz*, we can display this information as a bar chart.

Alternatives	Score
a01	26.2
a02	61.95
a03	40.45



6 – Outranking Methods

The concept of outranking relations was born with the intention to overcome some of the difficulties of the aggregation approaches based on MAUT. For example, MAUT methods are based on the concept of dominance relation and cannot deal with other types of relations such as incomparability. Moreover, the use of ordinal criteria is difficult in MAUT, but very natural in outranking.

This approach focuses the attention to the fact that in MCDA problems one tries to establish preference orderings of alternatives (17), (18). As each criterion usually leads to different ranking of the alternatives, the problem is to find a consensus ranking.

One of the most well-known outranking methods is ELECTRE (ELimination Et Choix Traduisant la RÉalité). Using the definition in (19), on outranking relation is a binary relation S defined in A such that aSb if, given what is known about the decision-maker's preferences and given the quality of the valuations of the actions and the nature of the problem, there are enough arguments to decide that a is at least as good as b, while there is no essential reason to refute that statement.

The technique uses factual information and/or subjective to assess simultaneously a set of alternatives *A* under different evaluation criteria *C*, that can be homogeneous, heterogeneous, quantified, qualified or mixture thereof. The way is through the use of preference relations, which compares the assessments $c(a_x)$ partially assigned to the alternatives a_x, identifying the best through a process of overrating.

The ELECTRE methods identify four preference situations concerning the comparison of two actions, using binary outranking relations, *S*, whose meaning is "*at least as good as*" (20):

- Indifference $\boldsymbol{I}$: it corresponds to a situation where there are clear and positive reasons that justify equivalence between the two actions (it leads to a reflexive and symmetric binary relation).
    - Ex: $\boldsymbol{aSb}$ and $\boldsymbol{bSa}$, $\boldsymbol{aIb}$ (a is indifferent to b).

- Strict Preference $\boldsymbol{P}$: it corresponds to a situation where there are clear and positive reasons in favour of one (identified) of the two actions (it leads to a no reflexive and asymmetric binary relation).
    - Ex: $\boldsymbol{aSb}$ and not $\boldsymbol{bSa}$, $\boldsymbol{aPb}$ (a is strictly preferred to b).
    - Ex: $\boldsymbol{bSa}$ and not $\boldsymbol{aSb}$, $\boldsymbol{bPa}$ (b is strictly preferred to a).
- Weak Preference $\boldsymbol{Q}$: it corresponds to a situation where there are clear and positive reasons that invalidate strict preference in favour of one (identified) of the two actions, but they are insufficient to deduce either the strict preference in favour of the other action or indifference between both actions, thereby not allowing either of the two preceding situations to be distinguished as appropriate (it leads to a no reflexive and asymmetric binary relation).
- Incomparability $\boldsymbol{R}$: it corresponds to an absence of clear and positive reasons that would justify any of the three preceding relations (it leads to a no reflexive and symmetric binary relation).
    - Ex: Not $\boldsymbol{aSb}$ and not $\boldsymbol{bSa}$, $\boldsymbol{aRb}$ (a is incomparable to b).

The construction of an outranking relation is based on two major concepts:

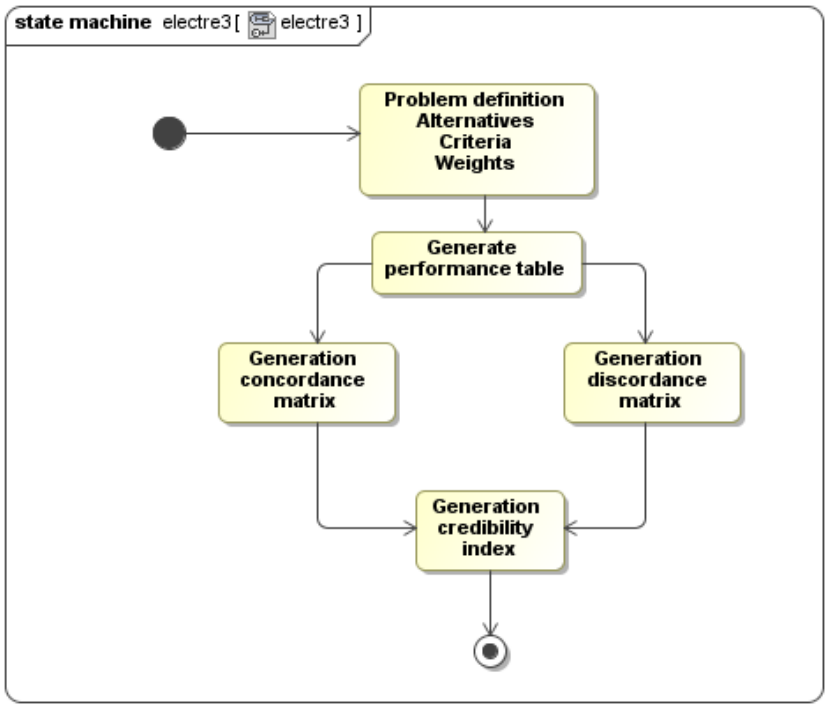
1. Concordance: For an outranking $\boldsymbol{aSb}$ to be validated, a sufficient majority of criteria should be in favour of this assertion.
2. Discordance: When the concordance condition holds, none of the criteria in the minority should oppose too strongly to the assertion $\boldsymbol{aSb}$.

From the concordance and discordance relations, a Credibility index is calculated, which corresponds to the value of "outranking" of a with regards to b. Different ways of implementing these relations lead to different versions of *ELECTRE*.

- **ELECTRE I**: it is the first method of outranking published. It is focused on solving Choice problems. The method reduces the number of alternatives forming a core of the best ones.
- **ELECTRE II**: it is a method also of outranking but more elaborated theoretically that the previous one. It allows obtaining a complete arrangement of the not dominated alternatives.
- **ELECTRE III**: in this method the relation of outranking is based on sets fuzzy, defining different thresholds for pseudo-criteria.

- **ELECTRE IV**: adapted for cases in which the decision maker does not want to specify the preferential weight.

In this document we will concentrate on explaining *ELECTRE III* and the procedure of the technology *Electre* consists of six steps like shows in the following figure, being described later.



6.1 – The ELECTRE III method

In *ELECTRE III* (21) the outranking relation can be interpreted as a fuzzy relation. The construction of this relation requires the definition of a *credibility index*, which characterizes the credibility of the assertion "$\boldsymbol{a}$ outranks $\boldsymbol{b}$", $\boldsymbol{aSb}$. It is defined by using both the concordance index, and a discordance index for each criterion c_j.

In this method, each criterion has three values associated to its description:

- q_j: Indifference threshold for a criterion c_j.
- p_j: Preference threshold for a criterion c_j.
- v_j: Veto preference for a criterion c_j.

Another important consideration of outranking methods is that the values in the performance table, $c_j\ a_x$, do not necessarily represent utility values. In fact, one of the main characteristics of this approach is that we can work with different performance scales in the different criteria, provided that they define a total order among the set of alternatives. In that sense, each criterion can be maximized or minimized, depending on its interpretation.

In the following sections, the procedures for calculating the concordance, discordance and credibility values are explained. The reader should take into account that when we are defining our criteria and alternatives, it is necessary to choose if we want to give him importance to the small values or to the big values. If we choose to give preference to the small values, we will have to calculate the difference between alternatives a_{x1} and a_{x2} in the following way:

$$Diff = -\ c_j\ a_{x1}\ - c_j\ a_{x2}$$

Otherwise, if we want to give preference to the high values, we will apply the following formula:

$$Diff = c_j\ a_{x1}\ - c_j\ a_{x2}$$

6.2 – Generation of the concordance matrix in ELECTRE III

To calculate the concordance matrix, we have to evaluate each pair of alternatives with their respective criteria. The preference p_j and indifference q_j thresholds are used to construct a concordance index ($Concordance_j\ a_{x1}, a_{x2}$) for each criterion, defined by:

$$Concordance_j\ a_{x1}, a_{x2} \quad \begin{matrix} 1 & If\ Diff\ \geq -q_j \\ 0 & If\ Diff\ \leq -p_j \\ \dfrac{Diff + p_j}{p_j - q_j} & else \end{matrix}$$

From the partial concordance, we calculate the overall concordance index:

$$Concordance\ a_{x1}, a_{x2}\ = \frac{w_j * Concordance_j\ a_{x1}, a_{x2}}{w_j}$$

6.3 – Generation of the discordance matrix in ELECTRE III

Discordance is defined similarly by the introduction of a veto threshold for each criterion, say v_j for criterion c_j, such the outranking of b by a is vetoed if the performance of b exceeds that of a by an amount greater than the veto threshold. A corresponding discordance index for each criterion is defined by as:

$$Discordance_j\ a_{x1}, a_{x2} \qquad \begin{matrix} 1 & If\ Diff\ \geq v_j \\ 0 & If\ \ Diff\ \leq p_j\ ||\ v_j \\ \dfrac{Diff - p_j}{v_j - p_j} & else \end{matrix}$$

6.4 – Generation of the credibility index in ELECTRE III

The credibility index is defined as follows:

$$S\ a_{x1}, a_{x2}\ = \begin{matrix} Concordance\ a_{x1}, a_{x2} & ,If\ D_j\ a_{x1}, a_{x2}\ \leq C\ a_{x1}, a_{x2}\ \ \forall j \\ Concordance\ a_{x1}, a_{x2} \prod_{j \epsilon J\ a_{x1}, a_{x2}} \dfrac{1 - D_j\ a_{x1}, a_{x2}}{1 - C\ a_{x1}, a_{x2}} & , otherwise \end{matrix}$$

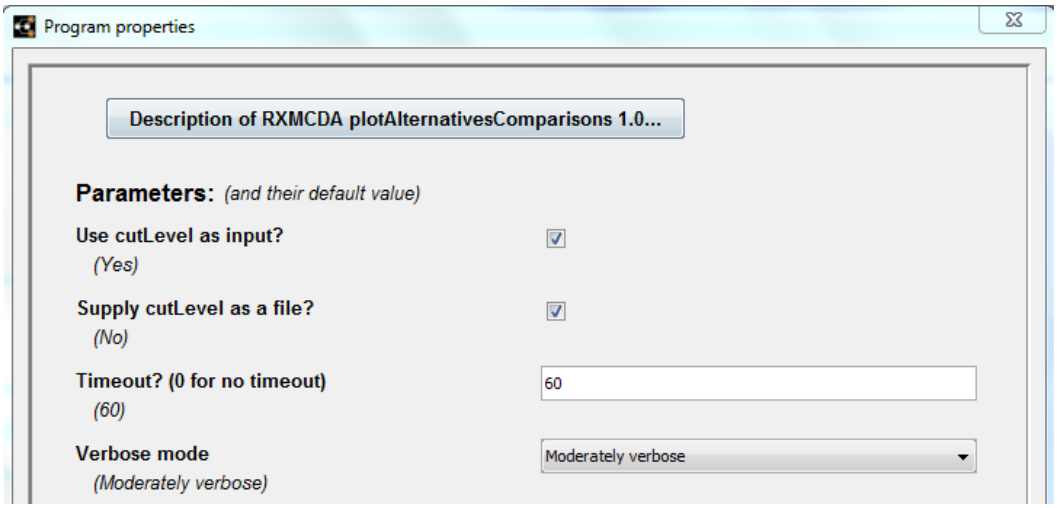
Where $J\ a_{x1}, a_{x2}$ is the set of criteria for which:

- $Discordance_{\mathrm{j}}\ \mathrm{a_{x1}}, \mathrm{a_{x2}} > Concordance\ \mathrm{a_{x1}}, \mathrm{a_{x2}}$.

6.5 – ELECTRE III in diviz

In this method we need to modify some of the input files used in the previous example, because *ELECTRE III* needs to include additional information. The files corresponding to the definition of *alternatives*, *PerformanceTable* and *Weights* are exactly the same as the case of WSM; however, in the file of criteria we will add the thresholds.

To display the results graphically we need a cut-off value, this value may be entered using a file (as we have done), or indicate it on the properties window associated to the corresponding modules, as illustrated below.



Graphic module properties

The data files are written using the language XMCDA. The following tables show the code of each data as an example. More information will be given in the next chapter. Moreover, a complete data file of each type is available in Annex B.

- **Alternatives:**

Code XMCDA

```
<alternatives>
   <alternative id="a01" name="Jyvaskyla sokos Hotel Alexandra" />
   <alternative id="a02" name="Jyvaskyla sokos hotel Jyvashovi" />
   <alternative id="a03" name="Cumulus Jyvaskyla" />
   <alternative id="a04" name="Scandic Jyvaskyla" />
   <alternative id="a05" name="Hotel Pension Kampus" />
   <alternative id="a06" name="Hotelli Alba" />
</alternatives>
```

- **Weights:**

Code XMCDA

```
<criteriaValues mcdaConcept="Importance" name="significance">
   <criterionValue>
     <criterionID>c01</criterionID>
     <value>
       <real>0.20</real>
     </value>
   </criterionValue>
   <criterionValue>
     <criterionID>c02</criterionID>
     <value>
       <real>0.30</real>
     </value>
   </criterionValue>
</criteriaValues>
```

- **Cut Level:**

Code XMCDA

```
<methodParameters>
   <parameter name="cutLevel">
     <value>
       <real>0.8</real>
     </value>
   </parameter>
</methodParameters>
```

- **Performance Table:**

Code XMCDA: only the first alternative

```
<performanceTable>
      <alternativePerformances>
            <alternativeID>a01</alternativeID>
            <performance>
                  <criterionID>c01</criterionID>
                  <value>
                        <real>1600.0</real>
                  </value>
            </performance>
            <performance>
                  <criterionID>c02</criterionID>
                  <value>
                        <real>300.0</real>
                  </value>
            </performance>
            <performance>
                  <criterionID>c03</criterionID>
                  <value>
                        <real>2.0</real>
                  </value>
            </performance>
            <performance>
                  <criterionID>c04</criterionID>
                  <value>
                        <real>3.0</real>
                  </value>
            </performance>
            <performance>
                  <criterionID>c05</criterionID>
                  <value>
                        <real>4.0</real>
                  </value>
            </performance>
            <performance>
                  <criterionID>c06</criterionID>
                  <value>
                        <real>5.0</real>
                  </value>
            </performance>
      </alternativePerformances>
</performanceTable>
```

- **Criterion:**

Code XMCDA: only the first criterion

```
<criteria>
      <criterion id="c01" name="DistCongres(m)">
            <scale>
                  <quantitative>
                        <preferenceDirection>min</preferenceDirection>
                  </quantitative>
            </scale>
            <thresholds>
                  <threshold mcdaConcept="ind">
                        <constant>
                              <real>200.0</real>
                        </constant>
                  </threshold>
                  <threshold mcdaConcept="pref">
                        <constant>
                              <real>700.0</real>
                        </constant>
                  </threshold>
                  <threshold mcdaConcept="veto">
                        <constant>
                              <real>1000.0</real>
                        </constant>
                  </threshold>
            </thresholds>
      </criterion>
</criteria>
```

7 – A case study: deciding about hotels

In this section we will see a case study of decision making, solved using the *Diviz* software. To carry out the case study we have chosen a real situation with real data.

In June 2011 it will be held the 21st International Conference on Multiple Criteria Decision Making (ICMCDM), and we need to find which is the best hotel, taking into account our budget limitations. The conference will take place in the city of Jyväskylä in Finland.

After making a filtering on the set of available hotels in Jyväskylä, we have found 6 hotels that fulfil the economic limitations. Therefore they are the alternatives that we will consider in this decision problem. The distribution of the hotels in the city is represented in the following figure.

For each hotel, 6 different characteristics will be considered to define our criteria. The description of the alternatives is given below.

1. **Name**: Jyväskylä sokos Hotel Alexandra.
   Street: Hannikaisenkatu,35, 40100.
2. **Name**: Jyväskylä sokos hotel Jyväshovi.

Street: Kauppakatu,35, 40100.

3. **Name**: Cumulus Jyväskylä.

   Street: Väinönkatu, 3, 40100.

4. **Name**: Scandic Jyväskylä.

   Street: Vapaudenkatu, 73, 40100.

5. **Name**: Hotel Pension Kampus.

   Street: Kauppakatu, 11 A 4, 40100.

6. **Name**: Hotelli Alba.

   Street: Ahlmaninkatu 4, 40100.

	Distance to the congress (m)	**Distance to the downtown (m)**	**Sports equipment**	**Restaurants**	**Stars**	**Services**
Hotel 1	1600	300	Sauna	3	4	Wi-Fi, Meeting room, ADSL
Hotel 2	1700	400	Sauna	2	4	Wi-Fi, Meeting room, ADSL
Hotel 3	1700	550	Swimming pool, sauna, gym	0	3	ADSL
Hotel 4	2000	350	Gym	2	4	Meeting room
Hotel 5	1200	110	No	0	1	No
Hotel 6	110	1300	No	1	3	Wi-Fi, meeting room
Weight	0.2	0.3	0.1	0.1	0.1	0.2
Goal	MIN	MIN	MAX	MAX	MAX	MAX

From this information we extract 6 indicators, which will be used to build the criteria corresponding to a particular decision maker, who is a researcher that wants to attend this conference.

His preferences are the following ones:

- The distance to the congress should be minimal, because if the hotel is far away from the conference site, one wastes a lot of time traveling. In addition, it is not possible to go to the hotel for a short rest during the day.
- The distance to the city centre is also a criterion to be minimized, however, it is not so important than the previous one. Being close to the touristic places in the city is desirable, but not a strict requirement.
- This person is very fond of sports. He prefers a hotel with pool and gym. Sauna is also a good option.
- The number of restaurants in the hotel is also important, because he likes to have different types of meals, and to be able to choose among different prices.
- The category of the hotel is going to be maximized, as a high quality hotel is always wishable.
- Finally, he requires some minimum services regarding mainly to Internet connection.

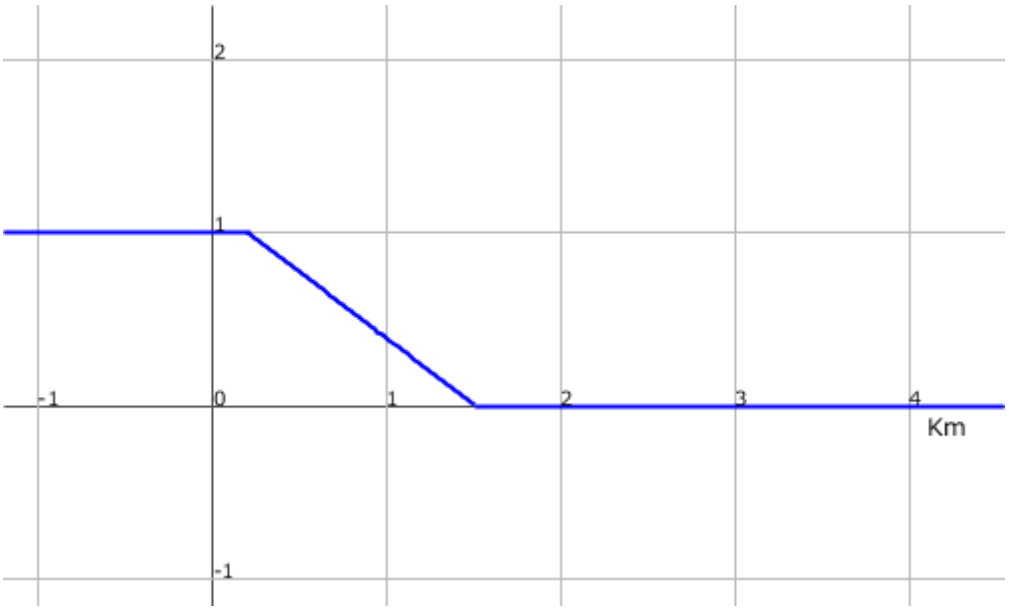
Now this problem of finding a suitable hotel for this person will be solved using the two decision models explained before, WSM and ELECTRE III.

7.1 – Solution with Weighted Sum in Diviz

As this method is based on the Utility Theory, we need to transform all the values of the different indicators into utility values. Moreover, we must take into account that we have to normalize the data using the same range of values. In this case we will use the 0-1 interval, where 0 means no performance (i.e. no utility for the decision maker) and 1 represents the complete fulfilment of the utility. Linear functions will be used to easily manage the utility.

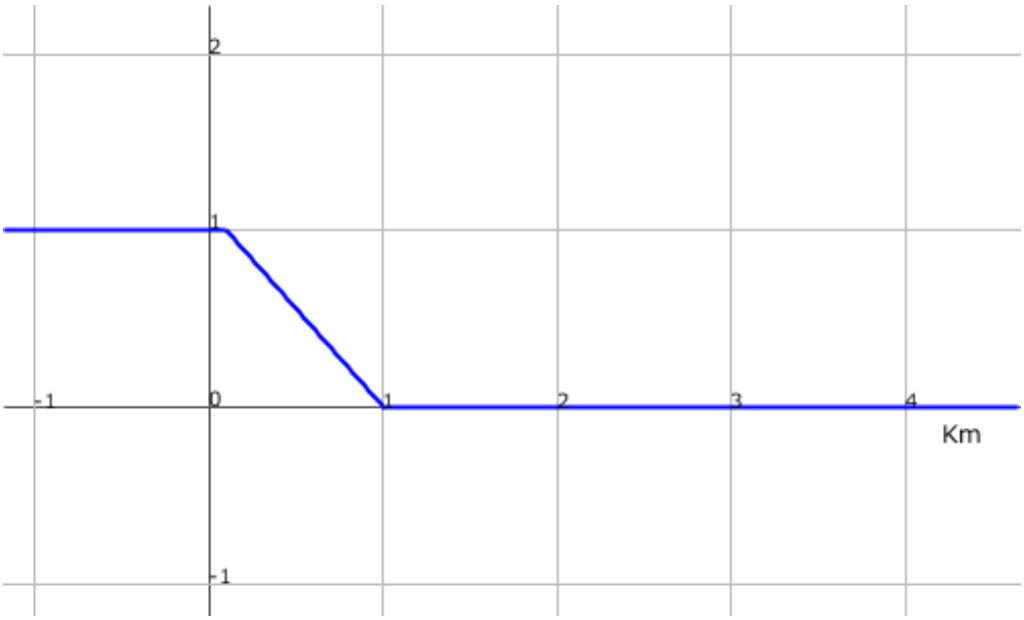
For the first criterion (distance to the conference), we have defined the following utility function:

$$x = \begin{cases} 1 & \forall x\ entre\ (-\infty, 0.2) \\ 0 & \forall x\ entre\ (1.5, \infty) \\ \dfrac{1.5 - x}{1.3} & \forall x\ entre\ (0.2, 1.5) \end{cases}$$



For the second criterion (distance to downtown), the utility function defined is as follows.

$$x = \begin{cases} 1 & \forall x\ entre\ (-\infty, 0.1) \\ 0 & \forall x\ entre\ (1, \infty) \\ \dfrac{1.9 - x}{0.9} & \forall x\ entre\ (0.1,1) \end{cases}$$



The rest of criteria are defined as a direct mapping between the set of possible values of the indicator, and the performance score that represents the user preferences. Also the 0-1 interval is used.

Sports equipment:

1. No → 0
2. Sauna → 0.5
3. Gym → 0.75
4. Swimming pool + Sauna + Gym → 1

Services:

1. No → 0
2. Meeting room → 0.5
3. ADSL → 0.5
4. Wi-Fi and meeting room → 0.75
5. Wi-Fi, meeting room and ADSL → 1

Restaurants:

1. 0 → 0
2. 1 → 0.75
3. 2 → 1
4. 3 → 1
5. 4 → 1

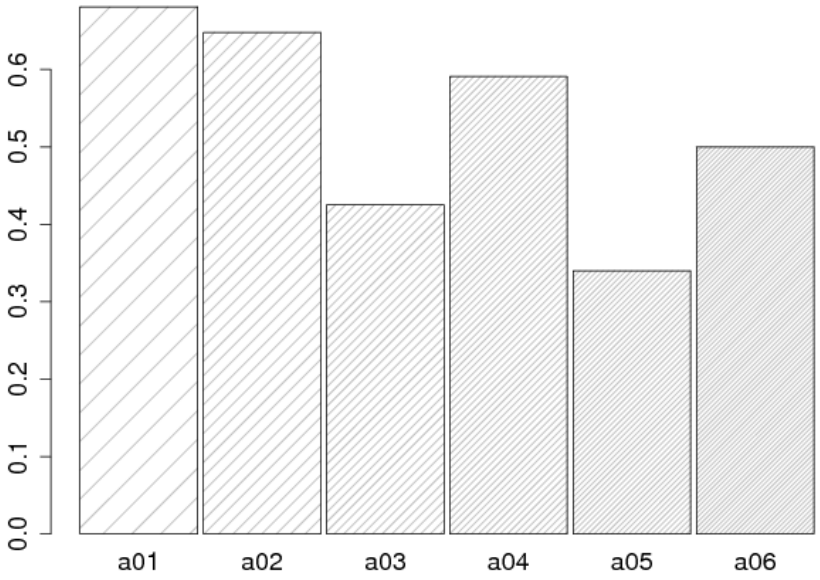
Starts:

1. 1 → 0
2. 2 → 0
3. 3 → 0.75
4. 4 → 1

After transforming the original data into utility values, the performance table obtained is the following one.

	Distance to the congress	**Distance to the downtown**	**Sports equipment**	**Restaurants**	**Stars**	**Services**
Hotel 1	0	0.77	0.5	1	1	1
Hotel 2	0	0.66	0.5	1	1	1
Hotel 3	0	0.5	1	0	0.75	0.5
Hotel 4	0	0.72	0.75	1	1	0.5
Hotel 5	0.23	0.98	0	0	0	0
Hotel 6	1	0	0	0.75	0.75	0.75
Weights	0.2	0.3	0.1	0.1	0.1	0.2

All these data has been modelled using XMCDA and then the *Diviz* WSM module has been executed. The results obtained are given below.



a01	0.68
a02	0.65
a03	0.42
a04	0.59
a05	0.34
a06	0.5

We can see that the hotel a_1 is the one that obtains the greatest utility score, although the difference with the hotel a_2 is quite small. Looking at their descriptions, we can observe that both hotels are near the city centre, they have different types of restaurants, are of high quality (4 stars) and have all the services facilities. They weak point is the distance to the congress, however, since we have used an additive model (which is compensatory), this drawback is compensated by the good performance in the rest of criteria. Moreover, notice that only one of the hotels is located near the congress (a_6), which also receives a good qualification, however, this hotel obtains quite low utility values in other criteria.

7.2 – Solution with ELECTRE III in Diviz

To solve the same problem of finding the most suitable hotel for the MCDM-2011 conference, we can use also an outranking approach. In this section, we explain the solution using ELECTRE III.

First of all, the criteria have been formulated according to the requirements of outranking methods, but always considering the same decision maker's preferences, detailed before.

In this case, the numerical attributes have been not modified. The categorical data has been transformed into a numerical scale of preference, according to the user's interests. The translation performed and the details about the different thresholds are given below.

Sports equipment:

1. No
2. Sauna
3. Gym.
4. Swimming pool + Sauna + Gym.

Services:

1. No
2. Meeting room.
3. ADSL.
4. Wi-Fi and meeting room.
5. Wi-Fi, meeting room and ADSL.

	Distance to the congress	Distance to downtown	Sports equipment	Restaurants	Stars	Services
Indif	200	100	0.0	0.0	1.0	1.0
Pref	700	300	1.0	1.0	2.0	3.0
Veto	1000	1000	3.0	3.0	3.0	4.0
Goal	MIN	MIN	MAX	MAX	MAX	MAX

The performance table corresponding to the 6 hotels in Jyväskylä, which is used as input in *ELECTRE III* is the following one:

	Distance to the congress	Distance to the downtown	Sports equipment	Restaurants	Stars	Services
Hotel 1	1600	300	2	3	4	5
Hotel 2	1700	400	2	2	4	5
Hotel 3	1700	550	4	0	3	3
Hotel 4	2000	350	3	2	4	2
Hotel 5	1200	110	1	0	1	1
Hotel 6	110	1300	1	1	3	4

7.2.1 – Generate the Concordance Matrix

In this section, we will explain in detail the different steps applied to generate the concordance matrix. Let us start with the calculation of the partial concordance between Hotel 4 with respect to Hotel 1. The values needed for this calculation are given in the following table.

	Distance to the congress	Distance to the downtown	Sports equipment	Restaurants	Stars	Services
Hotel 4	2000	350	3	2	4	2
Hotel 1	1600	300	2	3	4	5
Indif	200	100	0.0	0.0	1.0	1.0
Pref	700	300	1.0	1.0	2.0	3.0
Veto	1000	1000	3.0	3.0	3.0	4.0
Goal	MIN	MIN	MAX	MAX	MAX	MAX
Diff	-400	-50	1	-1	0	-3
Weights	0.2	0.3	0.1	0.1	0.1	0.2

The concordance calculation for each of the 5 criteria and hotels H4 and H1 are:

- $Concordance_1\ H_4, H_1\ = \frac{-400+700}{700-200} = 3\ _5 = 0.6, then$
- $Concordance_2\ H_4, H_1\ = \frac{-50+300}{300-100} = 250\ _{200} = 1, then$
- $Concordance_3\ H_4, H_1\ = 1,\ since\ 1 \geq 0$
- $Concordance_4\ H_4, H_1\ = 0,\ since - 1 \leq -1$
- $Concordance_5\ H_4, H_1\ = 1,\ since\ 0 \geq -1$
- $Concordance_6\ H_4, H_1\ = 0,\ since\ 3 \leq -3$

The overall concordance value for this pair of hotels is:

- $Concordance\ H_4, H_1\ = 0.6 * 0.2 + 1 * 0.3 + 1 * 0.1 + 1 * 0.1 = 0.62$

This process is repeated for all the pairs of hotels. The complete concordance matrix is:

	Hotel 1	Hotel 2	Hotel 3	Hotel 4	Hotel 5	Hotel 6
Hotel 1	1	1	0.9	0.9	0.78	0.8
Hotel 2	0.9	1	0.9	0.9	0.59	0.8
Hotel 3	0.57	0.72	1	0.75	0.58	0.7
Hotel 4	0.62	0.76	0.86	1	0.59	0.7
Hotel 5	0.5	0.5	0.7	0.7	1	0.4
Hotel 6	0.5	0.5	0.6	0.5	0.7	1

7.2.2 – Generate the Discordance matrix

To give an example of how to calculate the discordance value for a given pair of hotels, we will consider the following information.

	Distance to the congress	Distance to the downtown	Sports equipment	Restaurants	Stars	Services
Hotel 5	1200	110	1	0	1	1
Hotel 4	2000	350	3	2	4	2
Indif	200	100	0.0	0.0	1.0	1.0
Pref	700	300	1.0	1.0	2.0	3.0
Veto	1000	1000	3.0	3.0	3.0	4.0
Goal	MIN	MIN	MAX	MAX	MAX	MAX
Diff	-800	-240	2	2	3	1
Weights	0.2	0.3	0.1	0.1	0.1	0.2

The discordance calculation for the 5 criteria and hotels H5 and H4 is:

- $Discordance_1\ H_5, H_4\ = 0, since -800\ \leq 700$
- $Discordance_2\ H_5, H_4\ = 0, since -240 \leq 300$
- $Discordance_3\ H_5, H_4\ = \frac{2-1}{3-1} = 1\ _2 = 0.5, then$
- $Discordance_4\ H_5, H_4\ = \frac{2-1}{3-1} = 1\ _2 = 0.5, then$
- $Discordance_5\ H_5, H_4\ = 1,\ since\ 3 \leq 3$
- $Discordance_6\ H_5, H_4\ = 0,\ since\ 1 \leq 3$

In this case, no overall discordance value is calculated. So, the full set of discordance values are:

	Distance to the congress	Distance to the downtown	Sports equipment	Restaurants	Stars	Services
H1 R H1	0	0	0	0	0	0
H1 R H2	0	0	0	0	0	0
H1 R H3	0	0	0.5	0	0	0
H1 R H4	0	0	0	0	0	0
H1 R H5	0	0	0	0	0	0
H1 R H6	1	0	0	0	0	0
H2 R H1	0	0	0	0	0	0
H2 R H2	0	0	0	0	0	0
H2 R H3	0	0	0.5	0	0	0
H2 R H4	0	0	0	0	0	0
H2 R H5	0	0	0	0	0	0
H2 R H6	1	0	0	0	0	0
H3 R H1	0	0	0	1	0	0
H3 R H2	0	0	0	0.5	0	0
H3 R H3	0	0	0	0	0	0
H3 R H4	0	0	0	0.5	0	0
H3 R H5	0	0	0	0	0	0

H3 R H6	1	0	0	0	0	0
H4 R H1	0	0	0	0	0	0
H4 R H2	0	0	0	0	0	0
H4 R H3	0	0	0	0	0	0
H4 R H4	0	0	0	0	0	0
H4 R H5	0.33	0	0	0	0	0
H4 R H6	1	0	0	0	0	0
H5 R H1	0	0	0	1	1	1
H5 R H2	0	0	0	0.5	1	1
H5 R H3	0	0	1	0	0	0
H5 R H4	0	0	0.5	0.5	1	0
H5 R H5	0	0	0	0	0	0
H5 R H6	1	0	0	0	0	0
H6 R H1	0	1	0	0.5	0	0
H6 R H2	0	0.86	0	0	0	0
H6 R H3	0	0.64	1	0	0	0
H6 R H4	0	0.93	0.5	0	0	0
H6 R H5	0	1	0	0	0	0
H6 R H6	0	0	0	0	0	0

7.2.3 – Generate the Credibility index

From the overall concordance values and the discordance ones, we generate the credibility index. As an example we will consider the following information.

- $Discordance_2\ H_6, H_2\ = 0.86$
- $Concordance\ H_6, H_2\ = 0.5$

Then, the credibility index of the outranking relation for the hotels H6 and H2 is calculated as follows:

- $S\ H_6, H_2\ = 0.5 * \frac{1-0.86}{1-0.5} = 0.5 * \frac{0.14}{0.5} = 0.14$

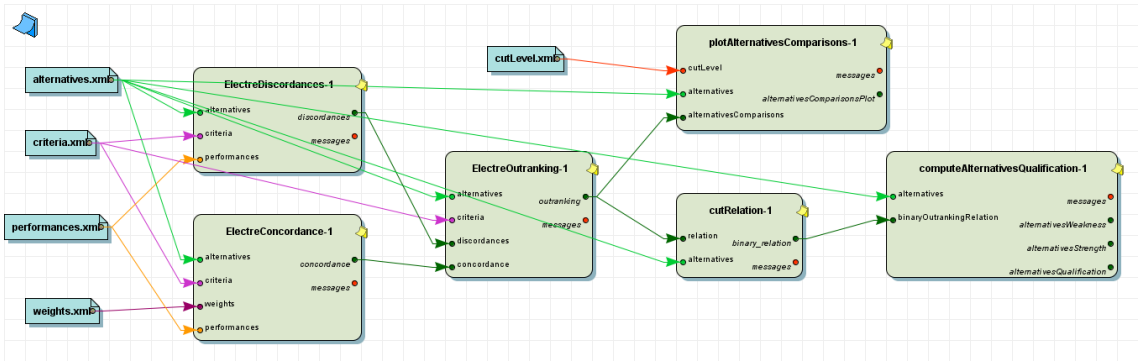
The complete credibility matrix is:

	Hotel 1	**Hotel 2**	**Hotel 3**	**Hotel 4**	**Hotel 5**	**Hotel 6**
Hotel 1	1	1	0.9	0.9	0.78	0
Hotel 2	0.9	1	0.9	0.9	0.59	0
Hotel 3	0	0.72	1	0.75	0.58	0
Hotel 4	0.62	0.76	0.8	1	0.59	0
Hotel 5	0	0	0	0	1	0
Hotel 6	0	0.14	0	0.07	0	1

7.2.4 – ELECTRE III in Diviz

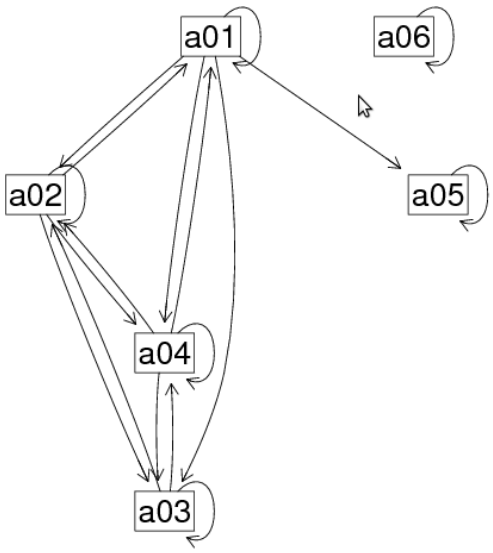
The data presented in the previous sections have been codified using the XCMDA language and introduced into the software *Diviz*. Complete code is available in Annex B.

The workflow of the model is given below. The result of outranking methods can be represented as a graph of outranking relations between the alternatives. In this case, we used a module to draw the result and display it clearly. This module needs a cut-off level for the values of credibility. Then, the outranking values that are below the cut threshold are considered as not relevant (getting a 0), and the rest correspond to the relevant outranking relations discovered (getting a value of 1).



With a cut-off of 0.6 the resulting credibility matrix is:

	Hotel 1	Hotel 2	Hotel 3	Hotel 4	Hotel 5	Hotel 6
Hotel 1	1	1	1	1	1	0
Hotel 2	1	1	1	1	0	0
Hotel 3	0	1	1	1	0	0
Hotel 4	1	1	1	1	0	0
Hotel 5	0	0	0	0	1	0
Hotel 6	0	0	0	0	0	1



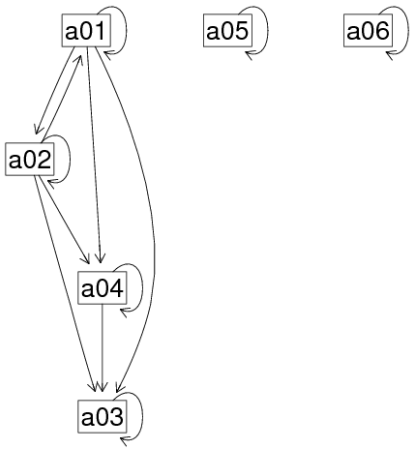
To find a ranking among the alternatives, two indicators are calculated: the Weakness (W) of ai corresponds to the number of alternatives that outrank ai, whereas the Strength (S) of ai is the number of alternatives that are outranked by ai. The difference S-W is the final qualification.

To choose the best hotel the S-W value is calculated from the previous graph:

	Weakness	**Strength**	**Qualification**
A01	3	5	2
A02	4	4	0
A03	4	3	-1
A04	4	4	0
A05	2	1	-1
A06	1	1	0

With a cut-off of 0.8 the resulting credibility matrix is:

	Hotel 1	**Hotel 2**	**Hotel 3**	**Hotel 4**	**Hotel 5**	**Hotel 6**
Hotel 1	1	1	1	1	0	0
Hotel 2	1	1	1	1	0	0
Hotel 3	0	0	1	0	0	0
Hotel 4	0	0	1	1	0	0
Hotel 5	0	0	0	0	1	0
Hotel 6	0	0	0	0	0	1



In this case, the S-W qualification is:

	Weakness	Strength	Qualification
A01	2	4	2
A02	2	4	2
A03	4	1	-3
A04	3	2	-1
A05	1	1	0
A06	1	1	0

We can see that hotel a_1 with the two cut-off values (0.6 and 0.8) is the one that get the highest score. If you look at the first result hotels a_2, a_4 and a_6 would be tied and a_5 and a_3 would be the worst.

In the second case, the cut-off is higher, so it eliminates some of the outranking relations because they are not so confident. Then, in second place we have hotel a_2 followed of a_5 and a_6 and in the last place a_3 and a_4. Notice, that in this second graph, a_5 and a_6 are incomparable with the rest of alternatives, having no outranking relation with them.

Annex A. WSM

Alternatives

```
<xmcda:XMCDA xmlns:xmcda="http://www.decision-deck.org/2009/XMCDA-
2.0.0" xmlns:xsi=http://www.w3.org/2001/XMLSchema-instance
xsi:schemaLocation="http://www.decision-deck.org/2009/XMCDA-2.0.0
http://sma.uni.lu/d2cms/xmcda/_downloads/XMCDA-2.0.0.xsd">
      <alternatives>
            <alternative id="a01">
                  <active>true</active>
            </alternative>
            <alternative id="a02">
                  <active>true</active>
            </alternative>
            <alternative id="a03">
                  <active>true</active>
            </alternative>
      </alternatives>
</xmcda:XMCDA>
```

Criteria

```
<xmcda:XMCDA xmlns:xmcda="http://www.decision-deck.org/2009/XMCDA-
2.0.0" xmlns:xsi=http://www.w3.org/2001/XMLSchema-instance
xsi:schemaLocation="http://www.decision-deck.org/2009/XMCDA-2.0.0
http://sma.uni.lu/d2cms/xmcda/_downloads/XMCDA-2.0.0.xsd">
<criteria>
      <criterion id="c01" />
      <criterion id="c02" />
      <criterion id="c03" />
      <criterion id="c04" />
</criteria>
</xmcda:XMCDA>
```

Performance table

```
<xmcda:XMCDA xmlns:xmcda="http://www.decision-deck.org/2009/XMCDA-
2.0.0" xmlns:xsi=http://www.w3.org/2001/XMLSchema-instance
xsi:schemaLocation="http://www.decision-deck.org/2009/XMCDA-2.0.0
http://sma.uni.lu/d2cms/xmcda/_downloads/XMCDA-2.0.0.xsd">
      <performanceTable>
            <alternativePerformances>
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                        <criterionID>c01</criterionID>
                        <value>
                              <integer>20</integer>
                        </value>
                  </performance>
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                        <criterionID>c02</criterionID>
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                              <integer>31</integer>
                        </value>
                  </performance>
```

```
      <performance>
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```

```
                        <criterionID>c04</criterionID>
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                              <integer>19</integer>
                        </value>
                  </performance>
            </alternativePerformances>
      </performanceTable>
</xmcda:XMCDA>
```

Weights

```
<?xml version='1.0' encoding='UTF-8'?>
<?xml-stylesheet type='text/xsl' href='xmcdaXSL.xsl'?>
    <criteriaValues mcdaConcept="weights">
        <criterionValue>
            <criterionID>c01</criterionID>
            <value>
                <real>0.20</real>
            </value>
        </criterionValue>
        <criterionValue>
            <criterionID>c02</criterionID>
            <value>
                <real>0.15</real>
            </value>
        </criterionValue>
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                <real>0.40</real>
            </value>
        </criterionValue>
        <criterionValue>
            <criterionID>c04</criterionID>
            <value>
                <real>0.25</real>
            </value>
        </criterionValue>
    </criteriaValues>
</xmcda:XMCDA>
```

Annex B. ELECTRE III

Alternatives

```
<xmcda:XMCDA xmlns:xmcda=http://www.decision-deck.org/2009/XMCDA-2.0.0
xmlns:xsi="http://www.w3.org/2001/XMLSchema-instance">
      <alternatives>
            <alternative id="a01" name="Jyvaskyla sokos Hotel
Alexandra" />
            <alternative id="a02" name="Jyvaskyla sokos hotel
Jyvashovi" />
            <alternative id="a03" name="Cumulus Jyvaskyla" />
            <alternative id="a04" name="Scandic Jyvaskyla" />
            <alternative id="a05" name="Hotel Pension Kampus" />
            <alternative id="a06" name="Hotelli Alba" />
      </alternatives>
</xmcda:XMCDA>
```

Criteria

```
<xmcda:XMCDA xmlns:xmcda=http://www.decision-deck.org/2009/XMCDA-2.0.0
xmlns:xsi="http://www.w3.org/2001/XMLSchema-instance">
      <criteria>
            <criterion id="c01" name="DistCongres(m)">
                  <scale>
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                        <threshold mcdaConcept="pref">
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                                    <real>700.0</real>
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                        <threshold mcdaConcept="veto">
                              <constant>
                                    <real>1000.0</real>
                              </constant>
                        </threshold>
                  </thresholds>
            </criterion>
            <criterion id="c02" name="DistCC(m)">
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                        <quantitative>
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```

```
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```
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            </criterion>
      </criteria>
</xmcda:XMCDA>
```

Performance table

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xmlns:xsi="http://www.w3.org/2001/XMLSchema-instance">
      <performanceTable>
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```

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```
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        </alternativePerformances>
    </performanceTable>
</xmcda:XMCDA>
```

Weights

```
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xmlns:xsi="http://www.w3.org/2001/XMLSchema-instance">

    <criteriaValues mcdaConcept="Importance" name="significance">
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```

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                        <real>0.30</real>
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            </criterionValue>
      </criteriaValues>
</xmcda:XMCDA>
```

Cut level

```
<xmcda:XMCDA xmlns:xmcda=http://www.decision-deck.org/2009/XMCDA-2.0.0
xmlns:xsi="http://www.w3.org/2001/XMLSchema-instance">

      <methodParameters>
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                  <value>
                        <real>0.8</real>
                  </value>
            </parameter>
      </methodParameters>

</xmcda:XMCDA>
```

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